NEUTRON COLLISION THEORY

M. Ragheb
3/30/2006

INTRODUCTION

We wish to analyze the process by which neutrons scatter upon collision with the nuclei of different materials. The intended use is in shielding, dosimetry, and criticality calculations. The energy loss per collision would characterize the properties of different energy moderating materials such as graphite, light water, and heavy water.

The kinematics of two-body collisions processes are best described using the Center of Mass system (CM), rather than the Laboratory (LAB) system of coordinates. The reason is that scattering is isotropic in the CM frame, and it is easier described in it.

We then introduce the concept of a microscopic and macroscopic neutron cross section and describe the use of compiled cross sections data to estimate collision rate densities and reaction rates.

RELATIONSHIPS BETWEEN VELOCITIES AND ENERGIES IN CM AND LAB SYSTEMS

The CM system is characterized by:

1. The total momentum in the CM system is zero.
2. The magnitudes of the CM velocities do not change in a collision. Their velocity vectors are rotated through the CM scattering angle.
3. Cross sections are calculated in the CM system, but are measured and used in the LAB system.
4. The total energy in the CM system is always less than in the LAB system. The energy difference is taken up by the center of mass' motion itself.

Let us consider:

Mass of target nucleus = A
Mass of neutron = 1
Target nucleus is stationary, implying that $\overline{V_L} = 0$

We can now deduce the relationships between velocities and energies in the CM and LAB systems.

The collision coordinates in the LAB and CM system before and after a collision are shown in Figs.1 and 2, as well as the relationships between the scattering angles in the LAB and the CM frames.

The Center of Mass velocity, is obtained by a momentum balance before and after a collision as:

$$(1+A)\overline{v_{CM}} = 1 \cdot \overline{v_L} + A \cdot \overline{V_L}$$

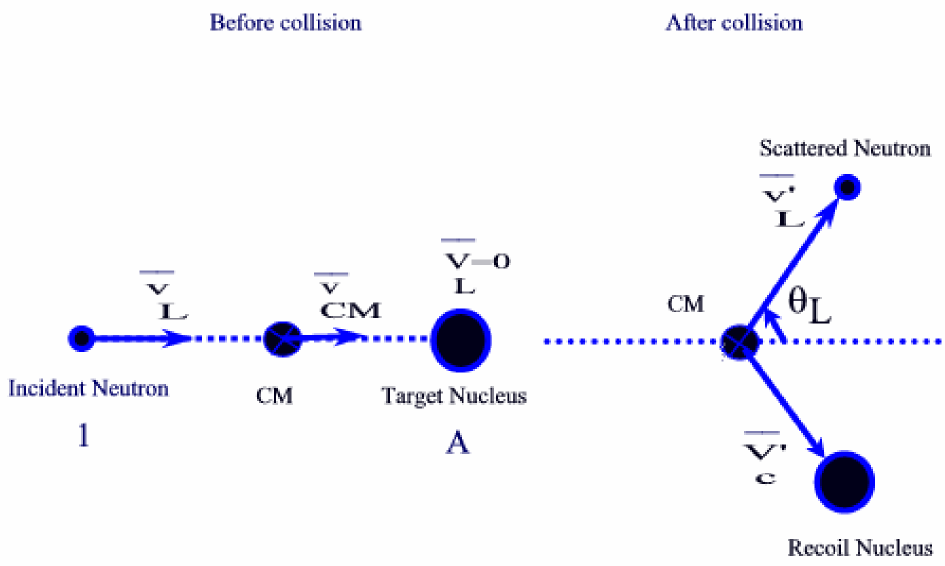

Fig.1 Collision parameters in Laboratory (LAB) system.

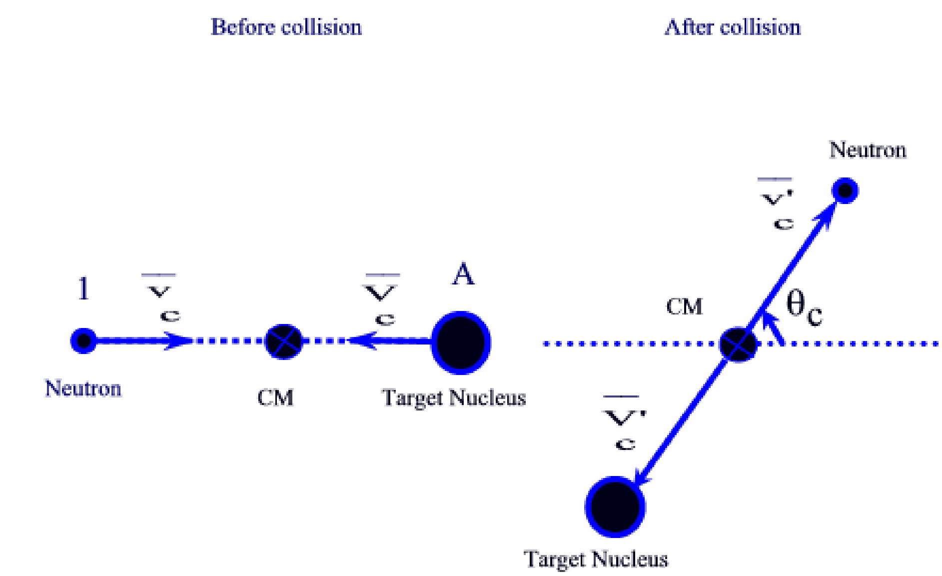

Fig.2 Collision parameters in Center of Mass (CM) system.

By taking $\overline{V_L} = 0$, we get:

$$\overline{v_{CM}} = \frac{1}{1+A}\overline{v_L} \tag{1}$$

The neutron velocity in the center of mass system using Eqn. 1 is:

$$\overline{v_C} = \overline{v_L} - \overline{v_{CM}} = \overline{v_L} - \frac{1}{1+A}\overline{v_L} \; .$$

From which:

$$\overline{v_C} = \frac{A}{1+A}\overline{v_L} \tag{2}$$

The target velocity in the Center of Mass system is from Eqn. 1:

$$\overline{V_C} = \overline{V_L} - \overline{v_{CM}} = -\frac{1}{1+A}\overline{v_L} \tag{3}$$

where again we took $\overline{V_L}$ = 0.

The total momentum in CM frame is by using Eqns. 2 and 3:

$$1 \cdot \overline{v_C} + A \cdot \overline{V_C} = 1 \cdot \frac{A}{1+A}\overline{v_L} - \frac{A}{1+A}\overline{v_L} = 0$$

The total kinetic energy in the LAB system is:

$$E_L = \frac{1}{2} \cdot 1 \cdot v_L^2 + \frac{1}{2} A V_L^2 = \frac{1}{2} v_L^2 \tag{4}$$

where $\overline{V_L}$ was taken as zero.

The total kinetic energy in the CM system is:

$$E_C = \frac{1}{2} \cdot 1 \cdot v_C^2 + \frac{1}{2} A V_C^2$$

Using Eqns.2 and 3 we get:

$$\begin{aligned} E_C &= \frac{1}{2} \cdot 1 \cdot \frac{A^2}{(A+1)^2} v_L^2 + \frac{1}{2} \frac{A}{(A+1)^2} v_L^2 \\ &= \frac{1}{2} \frac{A(A+1)}{(A+1)^2} v_L^2 \end{aligned}$$

Thus:

$$E_C = \frac{1}{2} \cdot \frac{A}{A+1} v_L^2 = \frac{1}{2} \mu v_L^2 \tag{5}$$

where:

$$\mu = \frac{A.1}{A+1} \quad ,$$

is the reduced mass.

The relationship between the LAB and CM velocities from Eqns 4 and 5 is:

$$E_C = \mu E_L = \frac{A}{A+1} E_L \tag{6}$$

Thus $E_C < E_L$, since the center of mass motion itself takes the energy difference.

Applying conservation of momentum in the CM system before and after a collision yields:

Before collision		After collision
$1 \cdot \overline{v}_c + A\overline{V}_c$	$=$	$A\overline{V}'_c + 1 \cdot \overline{v}'_c$

Rewriting this vector equation component-wise in the x and y directions:

$$v_C - AV_C = -AV'_C \cos\theta_c + v'_C \cos\theta_c \tag{7}$$

$$0 = -\, AV'_C \sin\theta_c + v'_C \sin\theta_c \tag{8}$$

Equation 8 implies that:

$$v'_C = AV'_C \tag{9}$$

Substituting in Eqn. 7 we get:

$$v_C = AV_C \tag{10}$$

Then using Eqn. 3 we get:

$$v_C = \frac{A}{1+A} V_L \tag{11}$$

Applying conservation of energy in the CM system yields:

$$\frac{1}{2}.1.v_C^2 + \frac{1}{2}AV_C^2 = \frac{1}{2}.1.v'^2_C + \frac{1}{2}AV'^2_C$$

Substituting for v_c and v'_c from Eqns.9 and 10:

$$\frac{1}{2}A^2V_C^2 + \frac{1}{2}AV_C^2 = \frac{1}{2}A^2V'^2_C + \frac{1}{2}AV'^2_C$$
$$(A+1)V_C^2 = (A+1)V'^2_C$$

which yields:

$$V_C = V'_C \qquad (12)$$

Substituting in Eqns. 9 and 10:

$$v_C = v'_C \qquad (13)$$

Thus the velocities do not change in the CM frame.

RELATIONSHIP BETWEEN SCATTERING CROSS SECTION IN LAB AND CM FRAMES

From Fig.3, we can write for the horizontal and vertical components:

Horizontal: $$v'_L \cos\theta_L = v_{CM} + v'_c \cos\theta_C \qquad (14)$$

Vertical: $$v'_L \sin\theta_L = v'_c \sin\theta_C \qquad (15)$$

Dividing the Left Hand Side (LHS) of both equations, we get:

$$\tan\theta_L = \frac{v'_C \sin\theta_c}{v_{CM} + v'_C \cos\theta_C} = \frac{\sin\theta_C}{\frac{1}{A} + \cos\theta_C} \qquad (16)$$

Here we used from Eqn.1: $$v_{CM} = \frac{1}{1+A}v_L$$

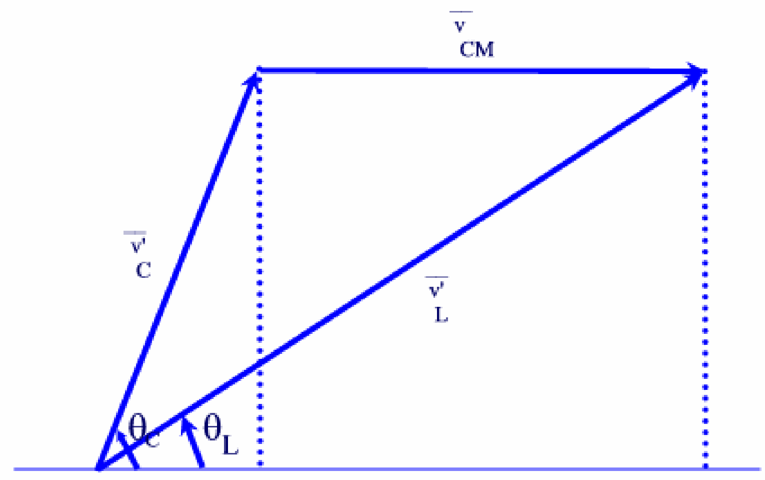

Fig.3 Relation between scattering angles in Center of mass (CM) and laboratory (LAB) systems.

and from Eqns.2,13:

$$v'_C = v_C = \frac{A}{1+A} v_L \; v'_C = v_C = \frac{A}{1+A} v_L$$

Now:

$$\sigma_{\mathrm{L}}(\theta_{\mathrm{L}}) \sin\theta_{\mathrm{L}} \cdot \mathrm{d}\theta_{\mathrm{L}} = \sigma_{\mathrm{CM}}(\theta_{\mathrm{C}}) \sin\theta_{\mathrm{C}} \cdot \mathrm{d}\theta_{\mathrm{C}}$$

where: $\sigma_{\mathrm{L}}(\theta_{\mathrm{L}})$ is the differential scattering cross section in the LAB system,

$\sigma_{\mathrm{CM}}(\theta_{\mathrm{C}})$ is the differential scattering cross section in the CM system.

Thus:

$$\sigma_{\mathrm{L}}(\theta_{\mathrm{L}}) = \sigma_{\mathrm{CM}}(\theta_{\mathrm{C}}) \cdot \frac{\sin\theta_{\mathrm{C}}}{\sin\theta_{\mathrm{L}}} \cdot \frac{\mathrm{d}\theta_{\mathrm{C}}}{\mathrm{d}\theta_{\mathrm{L}}} \tag{17}$$

From Eqns.15 and 13:

$$\frac{\sin\,\theta_{\mathrm{C}}}{\sin\,\theta_{\mathrm{L}}} = \frac{v'_L}{v'_C} = \frac{v'_L}{v_C}$$

Thus, from Eqn. 11:

$$\frac{\sin\ \theta_{\mathrm{C}}}{\sin\ \theta_{\mathrm{L}}} = \frac{v'_L}{\frac{A}{1+A} v_L} \tag{18}$$

Substituting in Eqn.17:

$$\sigma_{\mathrm{L}}(\theta_{\mathrm{L}}) = \sigma_{\mathrm{CM}}(\theta_{\mathrm{C}}) \cdot \frac{1+\mathrm{A}}{\mathrm{A}} \cdot \frac{v'_L}{v_L} \frac{\mathrm{d}\theta_{\mathrm{C}}}{\mathrm{d}\theta_{\mathrm{L}}} \tag{17)'}$$

Differentiating Eqn.16 with respect to θ_L, we get:

$$\sec^2\theta_{\mathrm{L}} = \frac{1}{\cos^2\theta_{\mathrm{L}}} = \frac{[(\frac{1}{A}+\cos\theta_{\mathrm{C}})\cos\theta_{\mathrm{C}} + \sin^2\theta_{\mathrm{C}}]}{(\frac{1}{\mathrm{A}}+\cos\theta_{\mathrm{C}})^2} \cdot \frac{\mathrm{d}\theta_{\mathrm{C}}}{\mathrm{d}\theta_{\mathrm{L}}}$$

Thus:

$$\frac{\mathrm{d}\theta_{\mathrm{C}}}{\mathrm{d}\theta_{\mathrm{L}}} = \frac{(\frac{1}{\mathrm{A}}+\cos\ \theta_{\mathrm{C}})^2}{\cos^2\theta_{\mathrm{L}}\ .(\frac{1}{A}\cos\theta_{\mathrm{C}}\ +1)} \tag{19}$$

To get an expression for $\cos^2\theta_L$ in terms of θ_C, we use Eqn.14:

$$\begin{aligned}\cos^2\theta_L &= (\frac{v_{CM}}{v'_L} + \frac{v'_C}{v'_L}\cos\theta_C)^2 \\ &= (\frac{1}{1+A}\frac{v_L}{v'_L} + \frac{A}{1+A}\frac{v_L}{v'_L}\cos\theta_C)^2\end{aligned}$$

by use of Eqns. 1, 11 and 13.

Thus:

$$\cos^2\theta_L = \frac{1}{(1+A)^2}\frac{v^2_L}{v'^2_L}(1+A\cos\theta_C)^2 \tag{20}$$

Substituting Eqn.20 into Eqn.19, we get:

$$\frac{d\theta_C}{d\theta_L} = \frac{(\frac{1}{A} + \cos\theta_C)^2}{\frac{A^2}{(1+A)^2}\cdot\frac{v_L^2}{v'^2_L}\cdot(\frac{1}{A} + \cos\theta_C)^2(\frac{1}{A}\cos\theta_C + 1)} = \frac{\frac{(1+A)^2}{A^2}\frac{v_L^2}{v'^2_L}}{(\frac{1}{A}\cos\theta_C + 1)} \tag{21}$$

Substituting Eqn.21 into Eqn. (17)':

$$\sigma_L(\sigma_L) = \sigma_{CM}(\sigma_C)\cdot\frac{(1+A)^3}{A^3}\cdot\frac{v'^3_L}{v_L^3}\cdot\frac{1}{(\frac{1}{A}\cos\theta_C + 1)} \qquad (17)''$$

To get the value of $(\frac{v'_L}{v_L})^3$ we use the triangular relationship from Fig. 3:

$$\begin{aligned} v'^2_L &= v_{CM}^2 + v'^2_C - 2v'_C v_{CM}\cos(180^o - \theta_C) \\ &= v_{CM}^2 + v'^2_c + 2v'_C v_{CM}\cos\theta_C \end{aligned}$$

Substituting for v_{CM} from Eqn.1, using $v'_C = v_C$, and substituting for v_L from Eqn.11, we get:

$$\frac{v'^2_L}{v_L^2} = \frac{1 + A^2 + 2A\cos\theta_C}{(1+A)^2} \tag{22}$$

Substituting Eqn.22 into Eqn. (17)", we get:

$$\sigma_L(\theta_L) = \sigma_{CM}(\theta_C)\cdot\frac{(1+A)^3}{A^3}\cdot\frac{1}{(1+A)^3}\cdot\frac{(1 + A^2 + 2A\cos\theta_C)^{3/2}}{(\frac{1}{A}\cos\theta_C + 1)}$$

Finally:

$$\sigma_L(\theta_L) = \sigma_{CM}(\theta_C)\frac{(\frac{1}{A^2} + \frac{2}{A}.\cos\theta_C + 1)^{3/2}}{(\frac{1}{A}\cos\theta_C + 1)} \tag{23}$$

which relates the scattering cross sections in the LAB and CM systems.

RELATIONSHIP BETWEEN THE INITIAL AND FINAL ENERGIES

To relate the final and initial particle energies in the LAB system, we use Eqn. 22:

$$\frac{E'}{E} = \frac{\frac{1}{2}.1.v'^2_L}{\frac{1}{2}.1.v_L^2} = \frac{A^2 + 1 + 2A\cos\theta_C}{(A+1)^2} \tag{24}$$

Defining the collision parameter:

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 \tag{25}$$

Thus:

$$1+\alpha = 2.\frac{A^2+1}{(A+1)^2}, \ \ 1\text{-}\alpha = 2.\frac{2A}{(A+1)^2}$$

And:

$$E' = [\frac{(1+\alpha)+(1-\alpha)\cos\theta_C}{2}]\cdot E \tag{26}$$

which relates the initial and final energies for a collision.

SPECIAL CHARACTERISTICS OF PARTICLES COLLISIONS ENERGY TRANSFER

Equation 26 describing the relationship between the initial and final energy of a neutron after collision with a nucleus possesses several important characteristics:

1. It implies that the energy transfer from neutron to nucleus is related to the scattering angle in the CM system.
   For the case of no collision we have $\theta_C = 0$, then:

   $$E' = E\ .$$

2. The maximum energy loss occurs in a back scattering collision: $\theta_C = 180^o$.
   Then:

$$E' = \alpha E .$$

The maximum energy loss would be:

$$\Delta E_{\max} = E - E' = E - \alpha E = (1-\alpha)E$$

3. Not only a neutron cannot gain energy in an elastics collision with a stationary nucleus (E'<E), but is cannot emerge with an energy E' less than the value of αE .

RELATIONSHIP BETWEEN THE SCATTERING ANGLES θ_C AND θ_L

By inspection of Fig.3 we can write using Eqns. 1 and 2:

$$\begin{aligned} v'_L \cos\theta_L &= v'_C \cos\theta_C + v_{CM} \\ &= \frac{A}{A+1} v_L \cos\theta_C + \frac{1}{A+1} v_L \end{aligned}$$

Substituting for v'_L from Eqn.22, we get:

$$v'_L = v_L \cdot \frac{(1+A^2+2A\cos\theta_C)^{1/2}}{(1+A)}$$

thus:

$$\frac{(1+A^2+2A\cos\theta_C)^{1/2}}{(1+A)} \cos\theta_L = \frac{A}{A+1}\cos\theta_C + \frac{1}{1+A}$$

So that:

$$\cos\theta_L = \frac{A\cos\theta_C + 1}{(A^2+2A\cos\theta_C+1)^{1/2}} \quad (27)$$

For high mass number elements such as Uranium, A>>1, the second term in the numerator, and the second and third terms in the denominator are small, then:

$$\cos\theta_L = \frac{A\cos\theta_C}{(A^2)^{1/2}} \approx \cos\theta_C$$

and the CM and LAB frames coincide to each other.

THE AVERAGE COSINE OF THE SCATTERING ANGLE $\overline{\mu_0}$

We can write in the LAB system:

$$\overline{\mu_0} = \overline{\cos\theta_L} = \frac{\int_0^{4\pi} \cos\theta_L d\Omega}{\int_0^{4\pi} d\Omega}$$

where:

$$d\Omega = 2\pi \sin\theta_C d\theta_C ,$$

is an element of solid angle.

Even though diffusion is isotropic in the CM frame, it is not so, in general, in the LAB frame.

The departure from isotropic scattering is measured in terms of $\overline{\cos\theta_L} = \overline{\mu_0}$, where:

$$\begin{aligned}
\overline{\mu_0} = \overline{\cos\theta_L} &= \frac{1}{4\pi}\int_0^{\pi} \cos\theta_L \cdot 2\pi \sin\theta_C d\theta_C \\
&= \frac{1}{2}\int_0^{\pi} \cos\theta_L \cdot \sin\theta_C d\theta_C \\
&= \frac{1}{2}\int_0^{\pi} \frac{A\cos\theta_C + 1}{(A2 + 2A\cos\theta_C + 1)^{1/2}} \sin\theta_C \cdot d\theta_C \\
&= \frac{1}{2}\int_0^{\pi} \frac{A\cos\theta_C + 1}{(A2 + 2A\cos\theta_C + 1)^{1/2}} d(\cos\theta_C)
\end{aligned}$$

$$\begin{aligned}
Let:\ \cos\theta_{\mathrm{C}} \equiv y, \theta_{\mathrm{C}} = 0 &\Rightarrow \cos\theta_{\mathrm{C}} = +1 \Rightarrow y = +1 \\
\theta_{\mathrm{C}} = \pi &\Rightarrow \cos\theta_{\mathrm{C}} = -1 \Rightarrow y = -1
\end{aligned}$$

Thus:

$$\begin{aligned}
\overline{\mu_0} &= \frac{1}{2}\int_{-1}^{+1} \frac{Ay + 1}{(A^2 + 2Ay + 1)^{1/2}} dy \\
&= \frac{1}{2}\left[\int_{-1}^{+1} \frac{Ay}{(A^2 + 2Ay + 1)^{1/2}} dy + \int_{-1}^{+1} \frac{1}{(A^2 + 2Ay + 1)^{1/2}} dy\right]
\end{aligned}$$

Now, from a table of integrals:

$$\int \frac{dx}{\sqrt{a + bx}} = \frac{2\sqrt{a + bx}}{b}, \quad \int \frac{xdx}{\sqrt{a + bx}} = -\frac{2(2a - bx)}{3b^2}\sqrt{a + bx}$$

Thus:

$$\overline{\mu_0} = \frac{1}{2}\left[\frac{A(-2)[2(A^2+1)-2Ay]Ay}{12A^2}\sqrt{(A^2+1)+2Ay}+\frac{2\sqrt{(A^2+1)+2Ay}}{2A}\right]_{-1}^{+1}$$

$$where: \quad \alpha \equiv \mathrm{A}^2+1, \quad b \equiv 2\mathrm{A}$$

$$\overline{\mu_0} = \frac{1}{2A}\left[-\frac{1}{3}\left[(A^2+1-A)(A+1)-(A^2+1+A)(A-1)\right]+(A+1)-(A-1)\right] \quad (28)$$

$$= \frac{1}{2A}\left[-\frac{1}{3}\left[(A^3+A-A^2+A^2+1-A-A^3-A-A^2+A^2+1+A)\right]+2\right]$$

$$= \frac{1}{2A}\left[-\frac{2}{3}+2\right] = \frac{1}{2A}\frac{6-2}{3} = \frac{2}{3A}$$

This relationship applies for elements other than hydrogen, that is for A not equal to unity.

THE SCATTERING PROBABILITY DISTRIBUTION FOR ELASTIC SCATTERING FROM STATIONARY NUCLEI

Since scattering is isotropic in CM system, the probability:

$$P(E')dE' = \frac{Number\ of\ \ favorable\ events\ scattering\ between\ \theta_{\mathrm{C}}\ and\ \ \theta_{\mathrm{C}}+d\theta_{\mathrm{C}}}{Total\ number\ of\ events},$$

can be expressed using Fig. 4 as:

$$P(E')dE' = \frac{Surface\ dS}{Total\ unit\ surface}$$

$$= \frac{2\pi\sin\theta_C d\theta_C}{4\pi\cdot 1^2}$$

$$= -\frac{d(\cos\theta_C)}{2}\cdot\frac{dE'}{dE'}$$

$$= -\frac{1}{2}\frac{d(\cos\theta_C)}{dE'}\cdot dE'$$

From Eqn. 26:

$$E' = \frac{(1+\alpha)+(1-\alpha)\cos\theta_C}{2} \cdot E$$

$$\cos\theta_C = \frac{2}{1-\alpha} \cdot \frac{E'}{E} - \frac{(1+\alpha)}{(1-\alpha)}$$

$$\frac{d(\cos\theta_C)}{dE'} = \frac{2}{(1-\alpha)E}$$

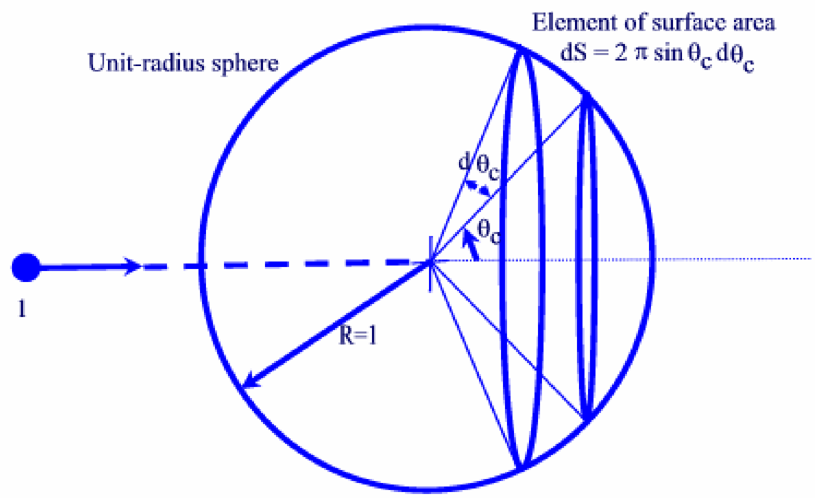

Fig.4 Geometry for elastic scattering in the Center of Mass (CM) system.

Thus:

$$P(E')dE' = -\frac{1}{(1-\alpha)E} \cdot dE' \qquad (29)$$

Since dE' is negative, P(E') is positive and equal to:

$$P(E') = \frac{1}{(1-\alpha)E}$$

and the collided neutron energy will be between E and αE . This is shown in Fig.5. The neutron has an equal probability of falling between the energies E'=E and E'=αE .

AVERAGE ENERGY OF NEUTRON AFTER A COLLISION

This, using Eqn. 29 can be written as:

$$
\begin{aligned}
\overline{E'} &= \int_{E}^{\alpha E} E' P(E') dE' \\
&= -\int_{E}^{\alpha E} E' \frac{1}{(1-\alpha)E} dE' \\
&= -\left[\frac{E'^2}{2}\right]_{E}^{\alpha E} \cdot \frac{1}{(1-\alpha)E} \\
&= \frac{E^2 - \alpha^2 E^2}{2} \frac{1}{(1-\alpha)E}
\end{aligned}
$$

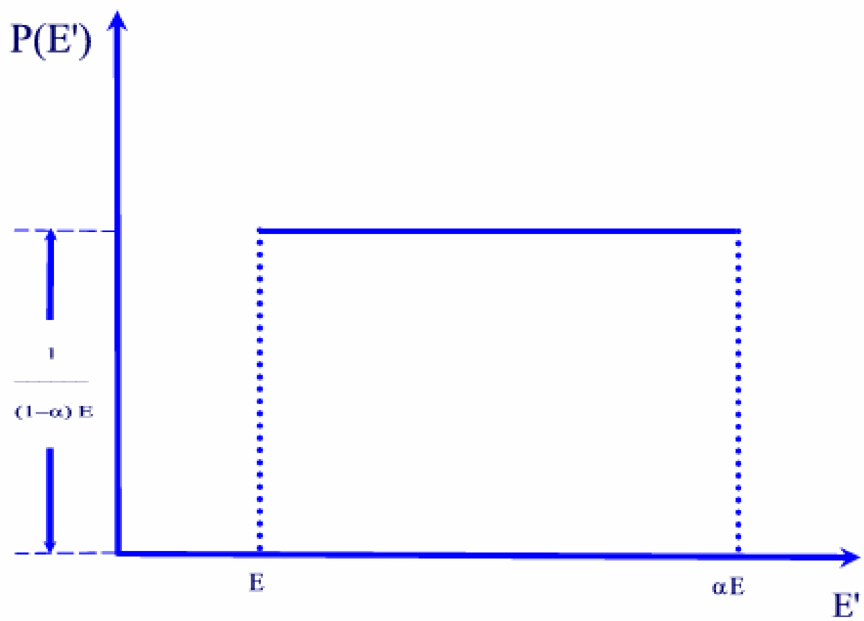

Fig. 5 Probabilty of scattering between dE' + E' and E'.

Thus:

$$\overline{E'} = (1+\alpha)\frac{E}{2} \tag{30}$$

THE AVERAGE LOGARITHMIC ENERGY DECREMENT PER COLLISION

The average value of decrease of the natural logarithm of neutron energy in a collision is:

$$\xi = \overline{\ln \frac{E}{E'}} = \overline{\ln E - \ln E'} \qquad (31)$$

Its utility is that it is independent of the neutron energy as shown below:

$$\xi = \int_{E}^{\alpha E} \ln \frac{E}{E'} P(E')dE'$$
$$= -\int_{E}^{\alpha E} \ln \frac{E}{E'} \cdot \frac{dE'}{(1-\alpha)E}$$

Making the change of variable:

$$\chi = \frac{E'}{E} \qquad \Rightarrow d\chi = \frac{dE'}{E}$$

$$\xi = \frac{-1}{1-\alpha} \int_{1}^{\alpha} \ln \frac{1}{\chi} \frac{Ed\chi}{E}$$
$$= +\frac{1}{1-\alpha} \int_{1}^{\alpha} \ln \chi d\chi$$
$$= \frac{1}{1-\alpha} [\chi \ln \chi - \chi]_{1}^{\alpha} = \frac{1}{1-\alpha} [\alpha \ln \alpha - \alpha + 1]$$

From which:

$$\xi = \frac{1}{1-\alpha} [1 - \alpha + \alpha \ln \alpha] = 1 + \frac{\alpha}{1-\alpha} \ln \alpha \qquad (32)$$

But the collision parameter is:

$$\alpha = \left(\frac{A-1}{A+1} \right)^2$$

Thus:

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \frac{(A-1)}{(A+1)} \qquad (33)$$

When A is large, (A>>1), for heavy elements:

$$\xi \rightarrow 1-2\frac{(A-1)^2}{2A}[\frac{1}{A}+\frac{1}{3A^3}+\frac{1}{5A^5}+...]$$

$$Since: \quad \ln(\frac{Z+1}{Z-1}) \approx 2(\frac{1}{Z}+\frac{1}{3Z^3}+\frac{1}{5Z^5}+...)$$

$$\xi = 1-\frac{A^2-2A+1}{A^2} \approx \frac{2}{A} \qquad (34)$$

by considering that the value of 1 in the numerator is small relative to the other terms.

For A>10 an expression correct to about 1% fitting experimental data is:

$$\xi = \frac{2}{A+\frac{2}{3}} \qquad (35)$$

In the case of mixture of elements in a moderator, the individual values of ξ are weighed by the scattering cross sections of each component to obtain its average value over the mixture:

$$\overline{\xi} = \frac{\sum_{i=1}^{n}\sigma_{si}\xi_i}{\sum_{i=1}^{n}\sigma_{si}} \qquad (36)$$

THE AVERAGE NUMBER OF COLLISIONS IN A MODERATOR

To slow down from energy E' E' to energy E'' E'' the number of neutron collisions can be estimated from:

$$N = \frac{\ln\frac{E'}{E''}}{\xi} \qquad (37)$$

SLOWING DOWN POWER AND MODERATING RATIOS

These moderator parameters are defined as:

$$\text{Slowing down power} = \sum_s \xi \qquad (38)$$

This is a measure of how efficient a material is in slowing-down the neutron energy.

$$\text{Moderating ratio} = \frac{\xi \sum_s}{\sum_a} \quad (39)$$

This is a measure of the efficiency of moderation without absorption.

Table 1 compares the values of the slowing down power and the moderating ratios for several materials. Deuterium used in heavy water distinguishes itself as a superior moderator. Nevertheless, carbon as graphite, light water and beryllium are also used as neutron moderators.

Table 1: Properties of major moderator materials.

Element	Mass Number A	Average Logarithmic Energy decrement ξ	Average Number of Collisions N	Macroscopic Absorption Cross section Σ_a	Slowing Down Power $\xi\Sigma_s$	Moderating Ratio $\xi\Sigma_s/\Sigma_a$
H	1	1	18	0.0792	1.53	72.0
D	2	0.725	25	0.0009	37.0	12,000.0
He	4	0.425	43	0.0	0.000016	83.0
Li	7	0.268	67	71.0	0.176	159.0
Be	9	0.209	80	0.008	-	-
B	11	0.176	103	780.0	-	170.0
C	12	0.158	115	0.005	0.064	-
O	16	0.120	150	0.0	-	-

THE LETHARGY OR LOGARITHMIC ENERGY DECREMENT

This is defined as:

$$u = \ln \frac{E_0}{E} \quad (40)$$

where E_0 is arbitrary reference energy corresponding to zero lethargy (e.g. 10 MeV).

The lethargy of a neutron increases as it is slowed down. The lethargy variable allows the expression of the neutron energy E as a dimensionless variable.

The lethargy change can be written as:

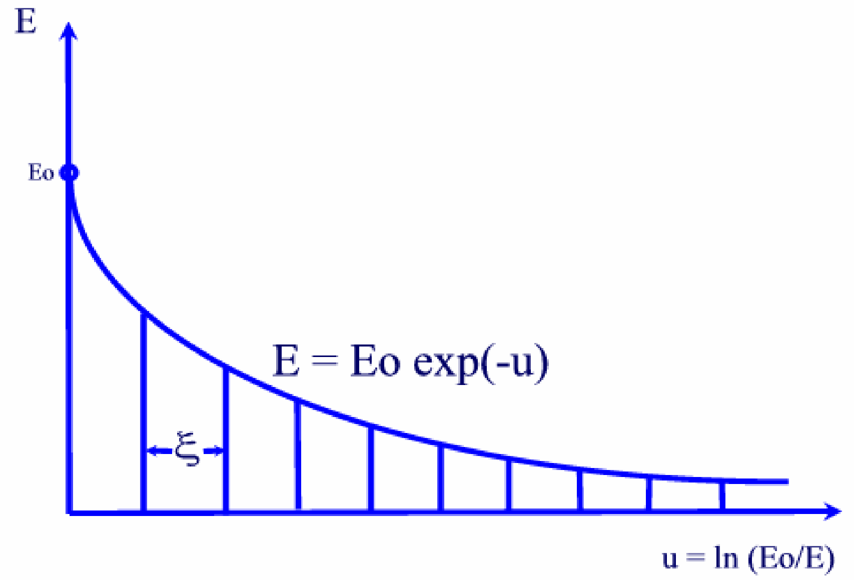

Fig. 6 Relationship between the energy and lethargy variables.

$$\Delta u = u_2 - u_1 = \ln \frac{E_1}{E_2} \quad (41)$$

From Eqn. 40:

$$E = E_0 e^{-u} \quad (42)$$

It is evident that ξ can be regarded as the average change in the lethargy of a neutron per collision. Regardless of its energy, a neutron suffers the same number of collisions for the same specified change in lethargy. Figure 6 shows that a neutron loses considerably more energy in earlier scatterings than in later ones.

REFERENCE

1. M. Ragheb, "Lecture Notes on Fission Reactors Design Theory," FSL-33, Department of Nuclear, Plasma and Radiological Engineering, 1982.

EXERCISE

1. Carry out the detailed derivation proving that, for elements other than hydrogen, the mean value of the cosine of the scattering angle for neutron collisions is given by:

$$\overline{\mu_0} = \overline{\cos\theta_L} = \frac{\int_0^{4\pi} \cos\theta_L d\Omega}{\int_0^{4\pi} d\Omega} = \frac{2}{3A} .$$